Interactive Mathematics Program

IMP

YEAR 1

Integrated High School Mathematics

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The Pit and the Pendulum

Excerpts from "The Pit and the Pendulum" by Edgar Allan Poe (1809-1849)

. . . Looking upward, I surveyed the ceiling of my prison. It was some thirty or forty feet overhead, and constructed much as the side walls. In one of its panels a very singular figure riveted my whole attention. It was the painted picture of Time as he is commonly represented, save that, in lieu of a scythe, he held what, at a casual glance, I supposed to be the pictured image of a huge pendulum such as we see on antique clocks. There was something, however, in the appearance of this machine which caused me to regard it more attentively. While I gazed directly upward at it (for its position was immediately over my own) I fancied that I saw it in motion. In an instant afterward the fancy was confirmed. Its sweep was brief, and of course slow. I watched it for some minutes, somewhat in fear, but more in wonder. Wearied at length with observing its dull movement, I turned my eyes upon the other objects in the cell. . . .

It might have been half an hour, perhaps even an hour, (for I could take but imperfect note of time) before I again cast my eyes upward. What I then saw confounded and amazed me. The sweep of the pendulum had increased in extent by nearly a yard. As a natural consequence its velocity was also much greater. But what mainly disturbed me was the idea that it had perceptibly *descended*. I now observed—with what horror it is needless to say—that its nether extremity was formed of a crescent of glittering steel, about a foot in length from horn to horn; the horns upward, and the under edge evidently as keen as that of a razor. Like a razor also, it seemed massy and heavy, tapering from the edge into a solid and broad structure above. It was appended to a weighty rod of brass, and the whole *hissed* as it swung through the air. . . .

What boots it to tell of the long, long hours of horror more than mortal, during which I counted the rushing oscillations of the steel! Inch by inch—line by line—with a descent only appreciable at intervals that seemed ages—down and still down it came! . . .

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The vibration of the pendulum was at right angles to my length. I saw that the crescent was designed to cross the region of the heart. It would fray the serge of my robe—it would return and repeat its operation—again—and again. . . .

Down—steadily down it crept. . . .

Down—certainly, relentlessly down! It vibrated within three inches of my bosom! . . .

I saw that some ten or twelve vibrations would bring the steel in actual contact with my robe, and with this observation there suddenly came over my spirit all the keen, collected calmness of despair. For the first time during many hours—or perhaps days—I *thought*. It now occurred to me, that the bandage, or surcingle, which enveloped me, was *unique*. I was tied by no separate cord. The first stroke of the razor-like crescent athwart any portion of the band, would so detach it that it might be unwound from my person by means of my left hand. But how fearful, in that case, the proximity of the steel! The result of the slightest struggle how deadly! Was it likely, moreover, that the minions of the torturer had not foreseen and provided for this possibility? Was it probable that the bandage crossed my bosom in the track of the pendulum? Dreading to find my faint, and, as it seemed, my last hope frustrated, I so far elevated my head as to obtain a distinct view of my breast. The surcingle enveloped my limbs and body close in all directions—*save in the path of the destroying crescent.*

Scarcely had I dropped my head back into its original position, when there flashed upon my mind what I cannot better describe than as the unformed half of that idea of deliverance to which I have previously alluded, and of which a moiety only floated indeterminately through my brain when I raised food to my burning lips. The whole thought was now present—feeble, scarcely sane, scarcely definite,—but still entire. I proceeded at once, with the nervous energy of despair, to attempt its execution.

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For many hours the immediate vicinity of the low framework upon which I lay, had been literally swarming with rats. They were wild, bold, ravenous; their red eyes glaring upon me as if they waited but for motionlessness on my part to make me their prey. "To what food," I thought, "have they been accustomed in the well?"

They had devoured, in spite of all my efforts to prevent them, all but a small remnant of the contents of the dish. I had fallen into an habitual see-saw, or wave of the hand about the platter; and, at length, the unconscious uniformity of the movement deprived it of effect. In their voracity the vermin frequently fastened their sharp fangs in my fingers. With the particles of the oily and spicy viand which now remained, I thoroughly rubbed the bandage wherever I could reach it; then, raising my hand from the floor, I lay breathlessly still.

At first the ravenous animals were startled and terrified at the change—at the cessation of movement. They shrank alarmedly back; many sought the well. But this was only for a moment. I had not counted in vain upon their voracity. Observing that I remained without motion, one or two of the boldest leaped upon the framework, and smelt at the surcingle. This seemed the signal for a general rush. Forth from the well they hurried in fresh troops. They clung to the wood—they overran it, and leaped in hundreds upon my person. The measured movement of the pendulum disturbed them not at all. Avoiding its strokes they busied themselves with the anointed bandage. They pressed—they swarmed upon me in ever accumulating heaps. They writhed upon my throat; their cold lips sought my own; I was half stifled by their thronging pressure; disgust, for which the world has no name, swelled my bosom, and chilled, with a heavy clamminess, my heart. Yet one minute, and I felt that the struggle would be over. Plainly I perceived the loosening of the bandage. I knew that in more than one place it must be already severed. With a more than human resolution I lay *still*.

Nor had I erred in my calculations—nor had I endured in vain. I at length felt that I was *free*. The surcingle hung in ribands from my body. But the stroke of the pendulum already pressed upon my bosom. It had divided the serge of the robe. It had cut through the linen beneath. Twice again it swung, and a sharp sense of pain shot through every nerve. But the moment of escape had arrived. At a wave of my hand my deliverers hurried tumultuously away. With a steady movement—cautious, sidelong, shrinking, and slow—I slid from the embrace of the bandage and beyond the reach of the scimitar. For the moment, at least, *I was free*.

The Question

The initial question of this unit is

Does the story's hero really have time to carry out his escape plan?

1. Based on the information you have, draw your own sketch of the prisoner's situation.
2. Next, go back through the story and carefully search for any additional information about the pendulum and the time for the prisoner's escape. Compile a group list of any information you find. If you are uncertain about the importance or relevance of a piece of information, write it down—you may need it later. Also write down any questions you have and identify any information you wish you had.
3. In your group, share your initial opinions about the question stated above.

Homework 1 Building a Pendulum

1. Tell the story of "The Pit and the Pendulum" to a family member, friend, or neighbor. Ask the listener if the time for the prisoner's escape plan seems realistic and how you could find out if it is.
2. Write about these topics.
   a. What were the reactions of your listener?
   b. Did the listener think the amount of time in the story seemed realistic? What was the listener's reasoning?
   c. How did the listener think you could find out how realistic this time estimate was?
3. Make a pendulum from materials that you find around your house.
4. Figure out a way to measure the period of your pendulum as accurately as you can. (Working with the family member, friend, or neighbor on this may be helpful.) *Remember:* The period is the time it takes for your pendulum to swing back *and* forth once.
5. Write about how you measured the period.
6. Bring your pendulum to class with you tomorrow.

Initial Experiments

You know that *something* determines the period of a pendulum, but it may not be clear to you exactly what that something is. Maybe there are several things.

In this activity, you will do some preliminary experiments to get an idea about what affects a pendulum's period.

1. Your teacher will assign you a variable from the list made in class. Do some experiments to see if that variable affects the period of a pendulum.
2. Prepare a written report, describing what you did, what observations you made, and what questions you still have.

Homework 2 Close to the Law

Zoe was doing a report on crime. Some people she interviewed believed that building more police stations would result in less crime. These people claimed that the closer one gets to a police station, the less crime there is.

Others thought that nearness to a police station was not an important factor in the level of crime in a neighborhood.

Zoe wanted to check out these competing claims. She called her local police station and got data on crimes in her area in the past year. She focused on robberies and how far each of the robberies had been from the station. She made this chart.

Number of Blocks from Police Station	Number of Crimes per Block
0–5	13
5–10	14
More than 10	16

1. Given this information, what relationship do you think there is between nearness to a police station and amount of crime? Explain your reasoning.
2. Zoe's brother Max thinks there might be other factors affecting crime rate besides distance from the police station. List at least three other factors that might account for the differences in crime rates in the table.

Pulse Gathering

If you repeatedly measure the same thing very carefully in the same way, will you get the same answer every time?

This activity provides a setting in which to look at this question.

1. Begin with your body at rest. Then count the pulse beats at your wrist for a 15-second interval. Record your result as a whole number of pulse beats.
2. Repeat step 1, again recording your result. Continue to repeat step 1 until you have ten results. (Some of these results may be identical.)
3. In preparation for *Homework 5: Pulse Analysis,* share your data with everyone else in your group, and record each other's results. In other words, you should have ten results for each person in the group (including yourself).

Homework 5 Pulse Analysis

You should have a collection of data on the number of pulse beats in a 15-second interval—ten results for yourself and ten for each of your fellow group members.

1. Did you get the same result each time you counted your pulse beats for a 15-second interval? Why or why not?

2. a. Find the mean (average) of your own pulse data.

   b. Find the mean of the pulse data for your whole group.

   c. Was the whole group's mean the same as your own? Why or why not?

3. For the following frequency bar graphs, do not group your measurements.

   a. Make a frequency bar graph of your own pulse data.

   b. Make a frequency bar graph for the pulse data of the whole group.

Homework 6 Return to the Pit

This unit is complex and includes some investigations that are not directly concerned with the main problem.

Therefore, from time to time in this unit, we will ask you to reflect on where you are with respect to solving the unit problem.

Write answers to these questions so that someone who knows nothing about the story and knows little about mathematics could understand what you are saying.

1. Write about the problem that the class is trying to solve, stating the goal as clearly as you can.

2. Write about what you have done so far in the unit and how that work will help to solve the unit problem. Be sure to explain clearly how measurement variation is involved.

3. Finally, write down some questions you have about the unit and some points you don't yet clearly understand.

Homework 7 What's Normal?

You've now seen some examples of normal curves. But when does the normal distribution apply to real life?

This assignment describes several situations. You may not know what the real information is, so just do your best. You might just make up a set of data that seems reasonable to you.

For each situation, follow these steps.

a. Draw a frequency bar graph of the situation based on your idea of what the data might look like. Your graph should show labeled axes and units of measurement, and you will need to decide on intervals that are suitable to the situation.

b. Explain how you decided what the graph should look like. If you guessed, explain what made you guess the way you did.

c. State whether or not your graph appears to be approximately a normal distribution.

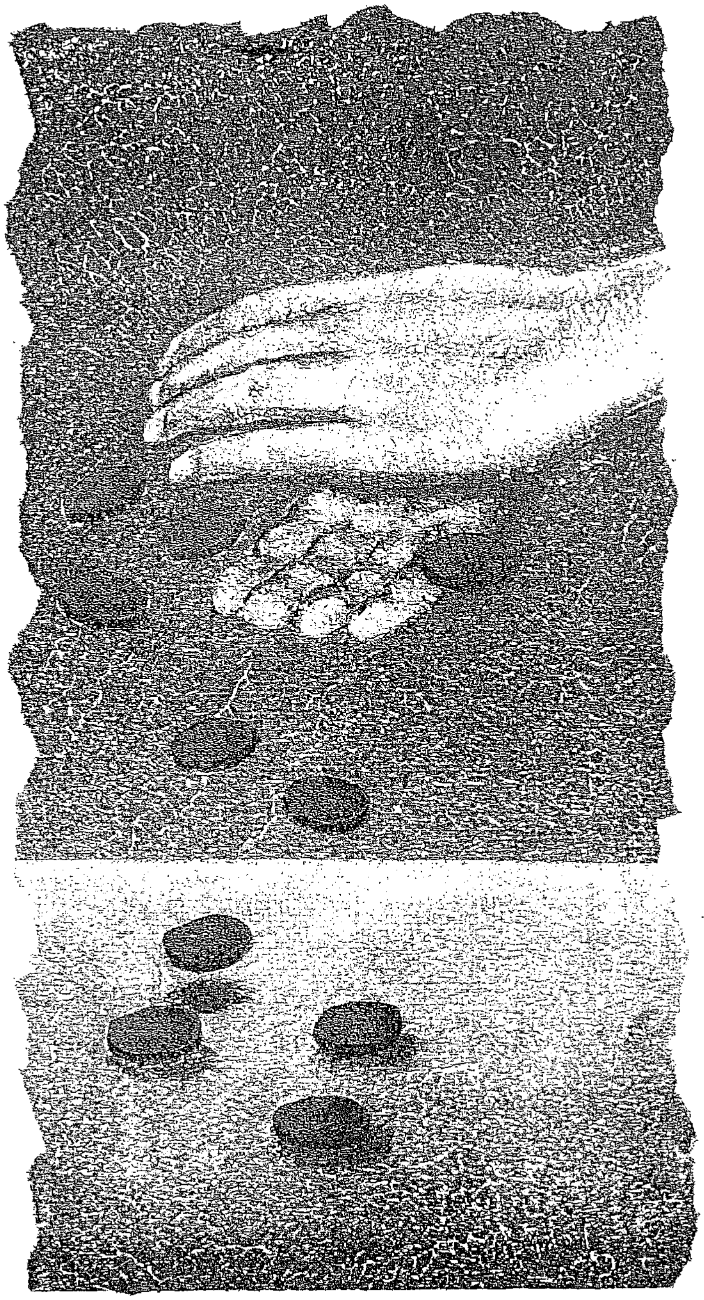
1. The number of people in your school who wear hightop tennis shoes, lowtop tennis shoes, dress shoes, or sandals to school on a given day.

2. The frequency with which a 100-meter sprinter achieves certain times, running 200 races over the course of one year. (Assume that the sprinter's average time is 12 seconds.)

3. The number of people in the United States who earn certain amounts of money. (Use 250,000,000 as the total population of the United States. You might use categories such as "income from $0 to $20,000," "income from $20,000 to $40,000," "income from $40,000 to $60,000," and so on.)

4. The number of people in the state of Hawaii who are of certain ages. (Use 1,100,000 as the total population of Hawaii.)

Homework 8 Flip, Flip



Do the results of coin flips give a bell-shaped distribution? You can get an idea by performing some experiments.

1. Shake 10 coins together and let them fall, and then record the number of heads. Do this experiment 15 times, recording the result each time, so that you get a total of 15 results, each of which is a number from 0 to 10.
2. Make a frequency bar graph showing the results of your experiments.
3. Do you think anyone in the class will have an experiment with a result of 0 (*no heads*)? with a result of 10 (*all heads*)? Explain your reasoning.
4. Predict what the class results will look like. That is, draw a frequency bar graph that you think will resemble the combined results from your class. Explain your reasoning.
5. Suppose you are given two coins and are told that *one of them* is unbalanced (but you don't know which one). You flip one of the coins 50 times, and it gives 28 heads and 22 tails. How confident would you be in deciding whether or not the coin you flipped is the unbalanced one? Explain your reasoning.

What's Rare?

Part I: Stride Lengths

Use the frequency bar graph of stride lengths to answer these questions.

1. Name a single stride length that you would categorize as *ordinary*, and name a single stride length that you would categorize as *rare*.

2. Where would you put the boundaries for each category? In other words, complete these sentences.

   a. An ordinary stride length is from -?- to -?-.

   b. A rare stride length is less than -?- or greater than -?-.

3. Based on your answers to Question 2, estimate the answer to the following questions.

   a. What percentage of all the data is in the "ordinary" category?

   b. What percentage of all the data is in the "rare" category?

Part II: Pulse Rates

Use the frequency bar graph of pulse rates to categorize these measurements as ordinary or rare for the pulse rate of a person at rest.

1. 20 beats for 15 seconds
2. 17 beats for 15 seconds
3. 12 beats for 15 seconds
4. 28 beats for 15 seconds

Continued on next page

5. Where would you place the borderline for each of the categories, ordinary and rare?

   a. An ordinary pulse rate is from -?- to -?-.

   b. A rare pulse rate is less than -?- or greater than -?-.

6. Based on your answers to Question 5, estimate the answers to the following questions.

   a. What percentage of all the data is in the "ordinary" category?

   b. What percentage of all the data is in the "rare" category?

Part III: Timing of Five Seconds

Use the frequency bar graph of the timing of five seconds to answer these questions.

1. Where would you place the borderline for each of the categories, ordinary and rare?

   a. An ordinary result is from -?- to -?-.

   b. A rare result is less than -?- or greater than -?-.

2. Based on your answers to Question 1, estimate the answers to the following questions.

   a. What percentage of all the data is in the "ordinary" category?

   b. What percentage of all the data is in the "rare" category?

Part IV: Comparing and Using Rarities

1. Compare the percentages you got in Question 2 of Part III with those you chose in Question 3 of Part I and Question 6 of Part II.

2. Suppose you got a new stopwatch, and used it to repeat the "timing of five seconds" experiments. If you found that you had an average of 5.7 seconds after 10 timings, would you think the new stopwatch was defective? What if your average after 10 timings with the new stopwatch was 4.9 seconds? Explain your reasoning.

Homework 9 Penny Weight

Sarah's and Tom's mom is a chemist, and one day she brought home a very sensitive scale. Sarah and Tom enjoyed learning how to use the scale.

One of the things they did was measure the weight of some pennies, one at a time. Here is a list of the results they got, arranged from lightest to heaviest (weights are in milligrams).

2600	2604	2607	2610	2612	2615	2616
2617	2618	2619	2623	2623	2624	2625
2626	2627	2630	2631	2636	2637	

1. Given this information, what do you think is the best estimate for the weight of a penny, and why?
2. Sarah's and Tom's Uncle Jack claimed that he had a counterfeit penny. Sarah and Tom didn't believe it was counterfeit, because it looked real and felt real and because their uncle was always trying to fool them. They asked him if they could borrow the penny, and they weighed it. They got 2641 milligrams.

   Tom said the coin must be counterfeit because they never got a weight that high with their other pennies. Sarah isn't sure. She thinks that if they weighed it again, its weight might be closer to that of the weight of the others. Or, if they measured more pennies, then Uncle Jack's coin might not seem so weird. What do you think, and why? If you don't think Uncle Jack's penny is counterfeit, then how heavy or light would a penny need to be before you believed it was counterfeit?

Mean School Data

Students at Kennedy and King High Schools were trying to determine what would affect the period of a pendulum.

At each school, students decided on standard pendulum characteristics for their initial experiments, including the length, weight, and amplitude.

Then five groups in each school took a fixed number of measurements of the period of this standard pendulum and calculated the mean for those measurements. The tables below give the mean pendulum periods found at each school.

Kennedy High

Group	Mean Pendulum Period (in seconds)
1	1.21
2	1.25
3	1.22
4	1.19
5	1.23

King High

Group	Mean Pendulum Period (in seconds)
1	1.16
2	1.22
3	1.31
4	1.11
5	1.30

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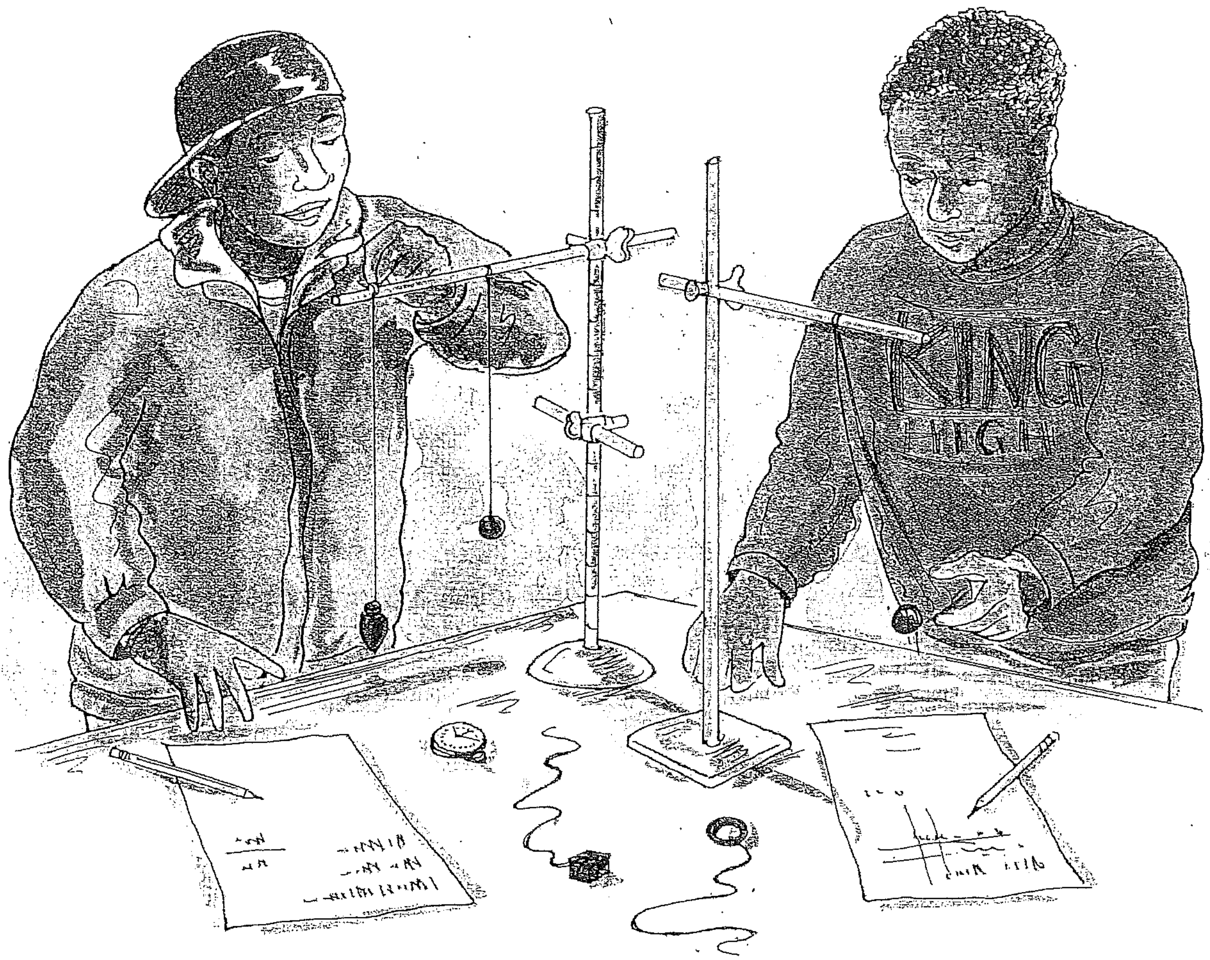
1. Find the overall mean for each school's data.

2. One group from each school decided to test whether changing the weight of the bob would change the period of a pendulum, so they conducted the set of experiments again with a different weight. Both groups now got a mean pendulum period of 1.29 seconds.

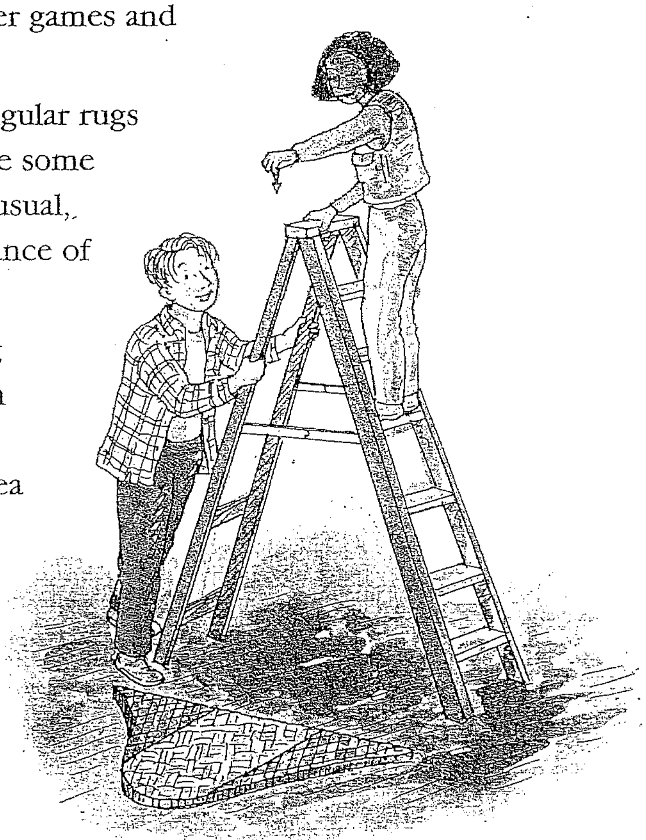
   If you were at Kennedy High, what would you conclude? If you were at King High, what would you conclude? In each case, explain your reasoning.



Homework 10 An (AB)Normal Rug

One day, Al and Betty got bored playing spinner games and decided to try rug and dart games.

Betty thought that playing on square or rectangular rugs would not be challenging enough, so she made some rugs that looked like normal distributions. As usual, each point in the rug had an equally likely chance of receiving a dart.

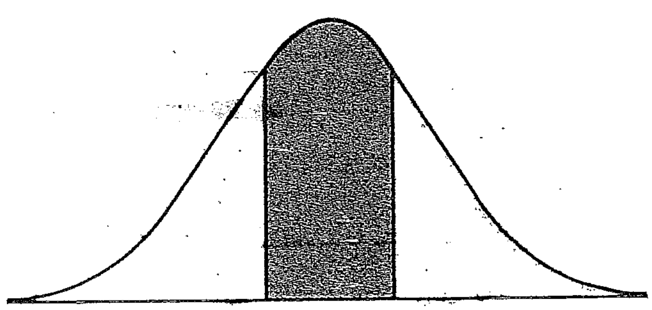


As shown in the diagram at the right, each rug consisted of three parts, with a central portion placed symmetrically between the other two. The central part of each rug resembled the area under the normal curve between two vertical lines symmetric around the mean. This part of the rug used one design. The two outer portions of the rug used a second design, and resembled the area under the normal curve to the left or right of such vertical lines.

Al and Betty then tried to guess where the dart would land. One guessed it would land in the central portion of the rug (the part using the first design), and the other guessed it would land outside this central part (in a portion of the rug using the second design).

1. The diagrams below are like the rugs used by Al and Betty, with the shaded area representing one design and the unshaded area representing the second design. In each case, estimate what percentage of the area is shaded.

a.



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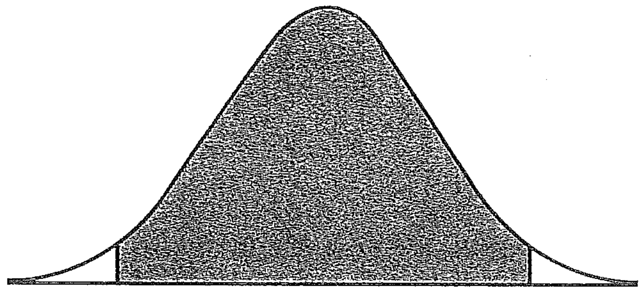
b.

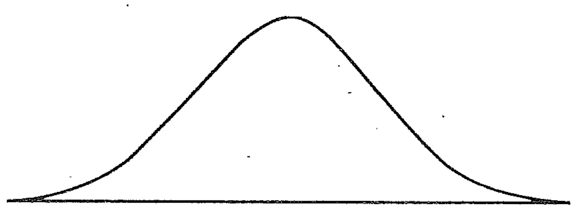


2. Trace each of the three rugs below. Then use two vertical lines to create regions like those above, to fit the condition in the next paragraph, and shade the region in the center. (Your shaded areas should each be centered around the vertical line of symmetry of the rug.)

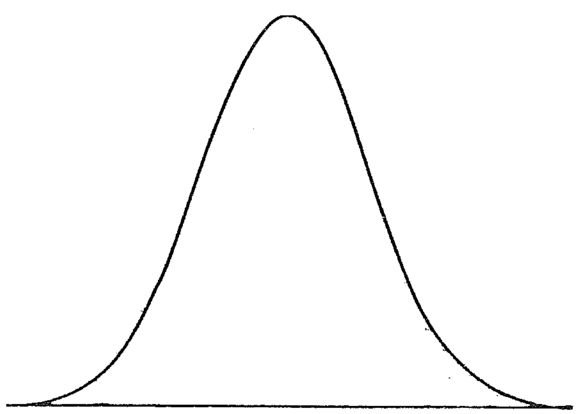
Your task is to estimate where to put the vertical lines so that a player who guesses that the dart will land in the shaded area wins twice as often as a player who guesses it will land in the unshaded area. Also, explain how you determined that one area is approximately twice as large as the other.

a.

b.

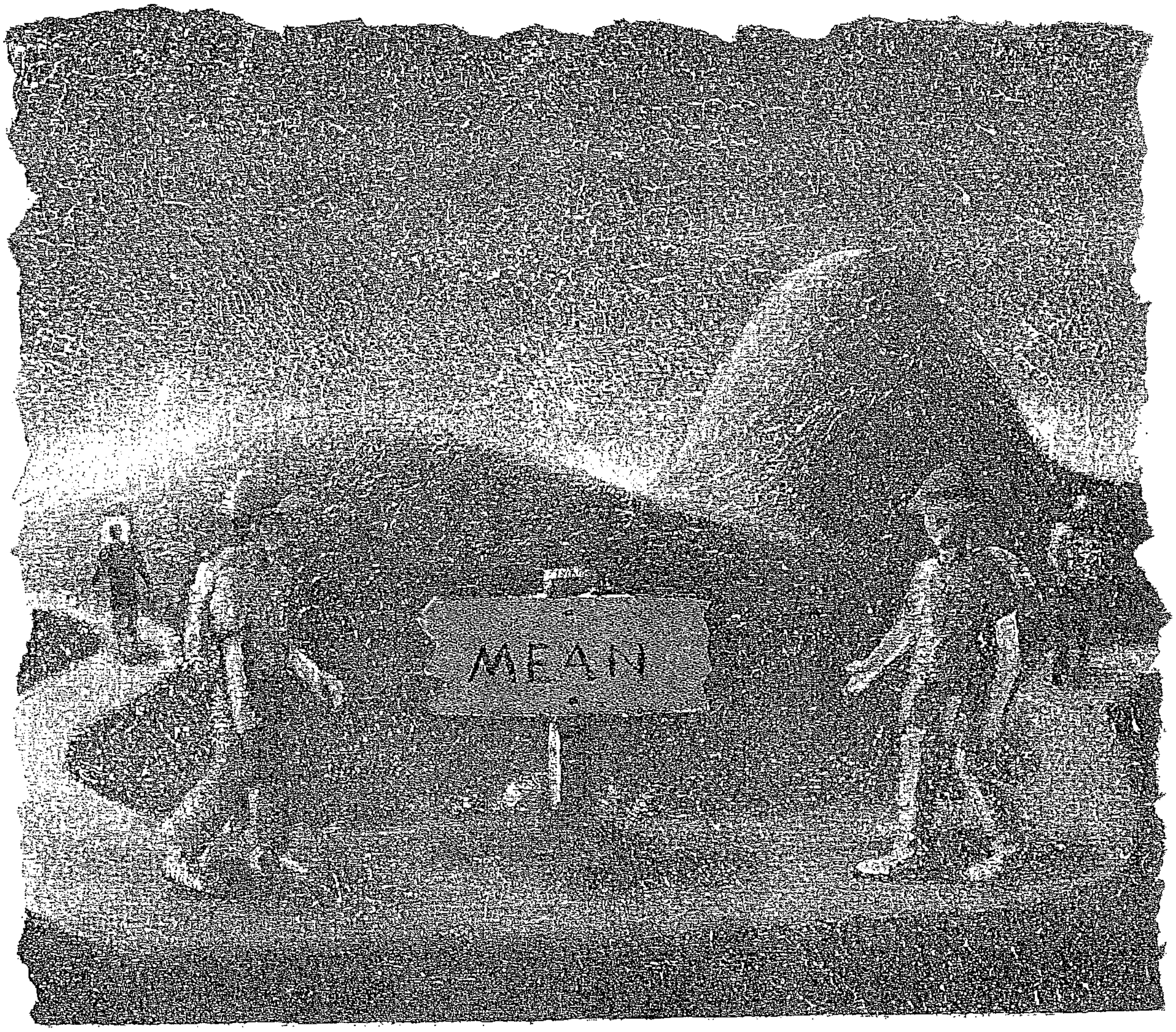


c.



3. Make another copy of the three rugs in Question 2, and repeat the process described there, except this time make the player with the shaded region win 95% of the time. Again, explain how you estimated the areas.

Data Spread



Rinky, Dinky, and Minky understood how to find the mean of any set of data. But they also knew that one set of data could have the same mean as another but look quite different.

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On one occasion, they looked at these four sets of data, each of which has a mean of 20.

Set A 19, 19, 20, 20, 21, 21

Set B 10, 10, 20, 20, 30, 30

Set C 12, 13, 13, 27, 27, 28

Set D 9, 20, 20, 20, 20, 31

1. Based on your own intuition, arrange the four sets of data from the set that is "least spread out from the mean" to the set that is "most spread out from the mean." Explain your reasons for the order you choose. (You and your group members may want to discuss this together, but each of you should make your own decision.)

Rinky, Dinky, and Minky were looking for a way to assign a number to measure how spread out from the mean a set of data was. They wanted a method in which the bigger the number, the more spread out the data set would be from its mean.

2. Rinky liked the idea of using the **range** of the data to measure data spread. To find the range, you just subtract the smallest number in the list from the largest one. For example, you find the range for set D by taking the difference $31 - 9$, which is 22.

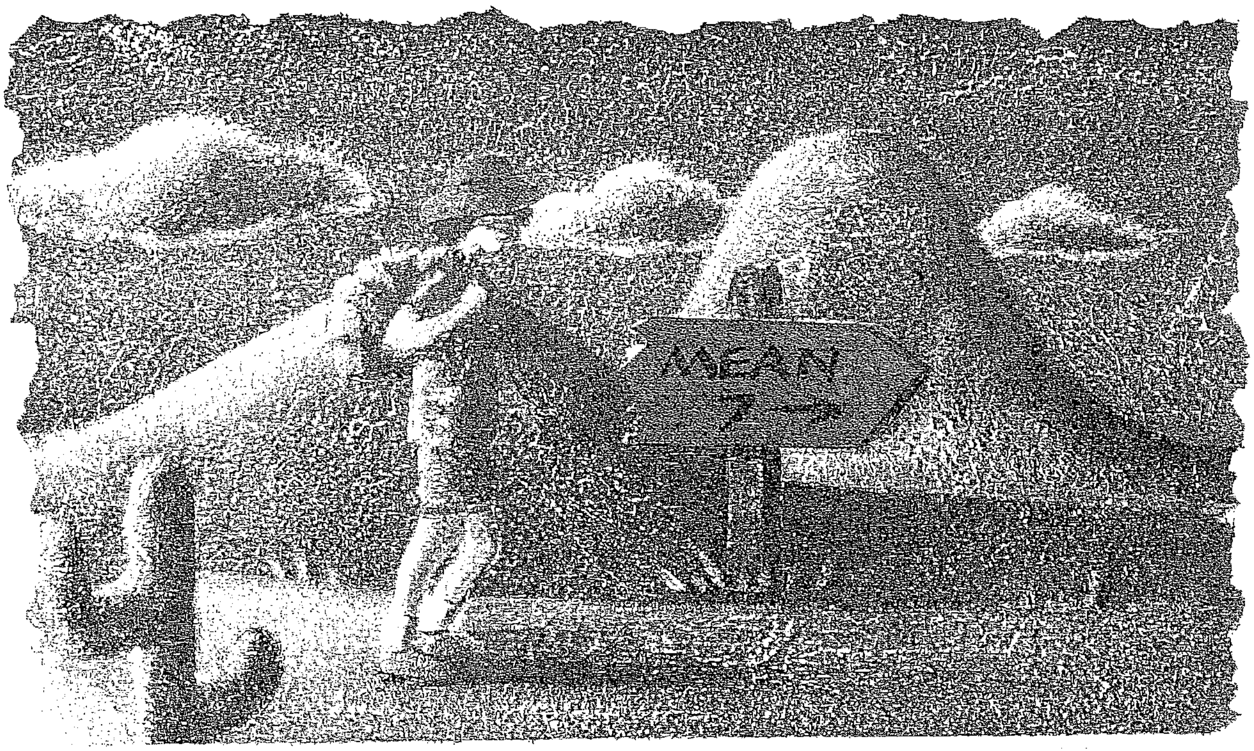
   a. Find the range for each of the other sets of data.

   b. Based on Rinky's method, arrange the four sets of data from the set that is "least spread out from the mean" to the set that is "most spread out from the mean."

   c. Does your result from Question 2b change your mind about your answer to Question 1? Explain.

Note: You will learn about Dinky's and Minky's ideas in *Homework 11: Dinky and Minky Spread Data.*

Homework 11 Dinky and Minky Spread Data



In *Data Spread,* you saw Rinky's idea for measuring data spread. His two friends had other suggestions.

For your convenience, here are the sets of data from that activity:

Set A	19, 19, 20, 20, 21, 21
Set B	10, 10, 20, 20, 30, 30
Set C	12, 13, 13, 27, 27, 28
Set D	9, 20, 20, 20, 20, 31

1. Dinky proposed finding the distance of each number in the list from the mean and then just adding those distances to get a measure for data spread.

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For example, in set *C*, because the mean is 20, the number 12 is 8 away from the mean. Similarly, each number 13 is 7 from the mean, and so on. So Dinky would assign the number $8 + 7 + 7 + 7 + 7 + 8$, which is 44, to set *C*.

a. Find the number that Dinky would assign to each of the other sets of data.

b. Based on Dinky's method, arrange the four sets of data from the set that is "least spread out from the mean" to the set that is "most spread out from the mean."

2. Minky's idea was to ignore the highest and lowest data items, removing just one item at each end even if there were ties. Then, he said, one should find the remaining data item that's farthest from the original mean and use that distance to measure data spread.

   For instance, with set *B*, Minky would drop the lowest number (one of the 10's) and the highest number (one of the 30's), leaving just 10, 20, 20, and 30.

   Because the mean of set *B* is 20, the maximum distance from any of these numbers to the mean is 10. So Minky would assign the number 10 to set *B*.

   a. Find the number that Minky would assign to each of the other sets of data.

   b. Based on Minky's method, arrange the four sets of data from the set that is "least spread out from the mean" to the set that is "most spread out from the mean."

3. Examine your answers to Questions 1b and 2b, as well as the answer to Question 2b of *Data Spread.* Whose measure of data spread—Rinky's, Dinky's, or Minky's—is closest to the answer you gave in Question 1 of *Data Spread*? Explain your decision.

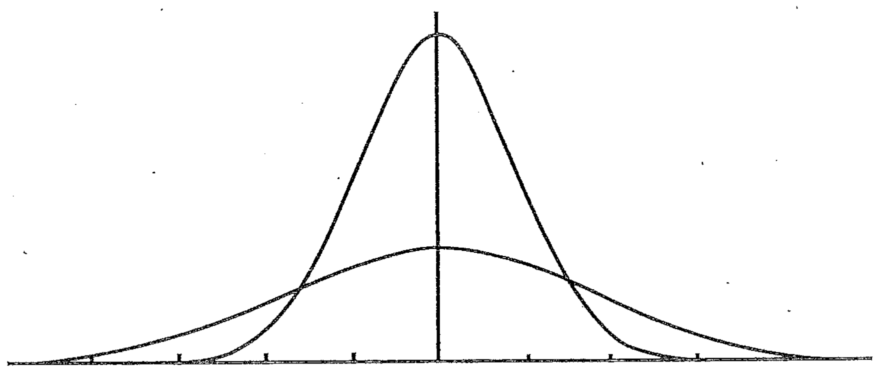
4. Invent a way to measure data spread that is different from these three. Describe how it works, and explain whether or not you think it is better or not.

Standard Deviation Basics

What Is Standard Deviation?

The **standard deviation** of a set of data measures how "spread out" the data set is. In other words, it tells you whether all the data items bunch around close to the mean or if they are "all over the place."

The superimposed graphs below show two normal distributions with the same mean, but the taller graph is less "spread out." Therefore, the data represented by the taller graph has a smaller standard deviation.



Calculation of Standard Deviation

Here is a list of the steps for calculating standard deviation.

1. Find the mean.
2. Find the difference between each data item and the mean.
3. Square each of the differences.
4. Find the average (mean) of these squared differences.
5. Take the square root of this average.

Organizing the computation of standard deviation into a table like the one on the next page can be very helpful. This table is based on a data set of five items: 5, 8, 10, 14, and 18. The mean for this data set is 11. The mean of a set of data is often represented by the symbol $\bar{x}$, which is read as "x bar."

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The computation of the mean is shown below the table to the left. On the right below the table, step 4 of the computation of the standard deviation is broken down into two substeps: (a) adding the squares of the differences and (b) dividing by the number of data items.

The symbol usually used for standard deviation is the lower case form of the Greek letter *sigma*, written σ.

x	$x - \bar{x}$	$(x - \bar{x})^2$
5	-6	36
8	-3	9
10	-1	1
14	3	9
18	7	49

sum of the data items = 55

number of data items = 5

$\bar{x}$ (mean of the data items) = 11

sum of the squared differences = 104

mean of the squared differences = 20.8

σ (standard deviation) = $\sqrt{20.8} \approx 4.6$

Suppose you represent the mean as $\bar{x}$, use n for the number of data items, and represent the data items as x_1, x_2, and so on. Then the standard deviation can be defined by the equation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}$$

Standard Deviation and the Normal Distribution

The normal distribution was identified and studied initially by a French mathematician, Abraham de Moivre (1667–1754). De Moivre used the concept of normal distribution to make calculations for wealthy gamblers. That was how he supported himself while he worked as a mathematician.

But the normal distribution applies to many situations besides those that are of interest to gamblers. (Measurement variation is one important example.) Therefore mathematicians have studied this distribution extensively.

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When we use standard deviation to study the variation among measurements of a pendulum's period, we make this assumption:

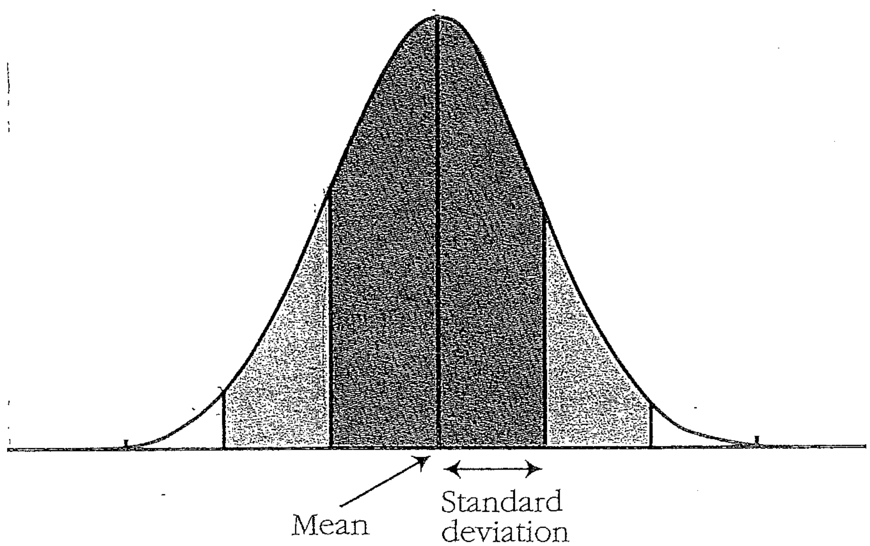
Normality Assumption

If you make many measurements of the period of any given pendulum, the data will closely fit a normal distribution.

One of the reasons why standard deviation is so important for normal distributions is that there are some principles about standard deviation that hold true for any normal distribution. Specifically, whenever a set of data is normally distributed, these statements hold true.

- Approximately 68% of all results are within one standard deviation of the mean.
- Approximately 95% of all results are within two standard deviations of the mean.

These facts can be explained in terms of area, using the diagram "The Normal Distribution."



The Normal Distribution

In this diagram, the darkly shaded area stretches from one standard deviation below the mean to one standard deviation above the mean; it is approximately 68% of the total area under the curve.

The light and dark shaded areas together stretch from two standard deviations below the mean to two standard deviations above the mean, and constitute approximately 95% of the total area under the curve.

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So standard deviation provides a good rule of thumb for deciding whether something is "rare."

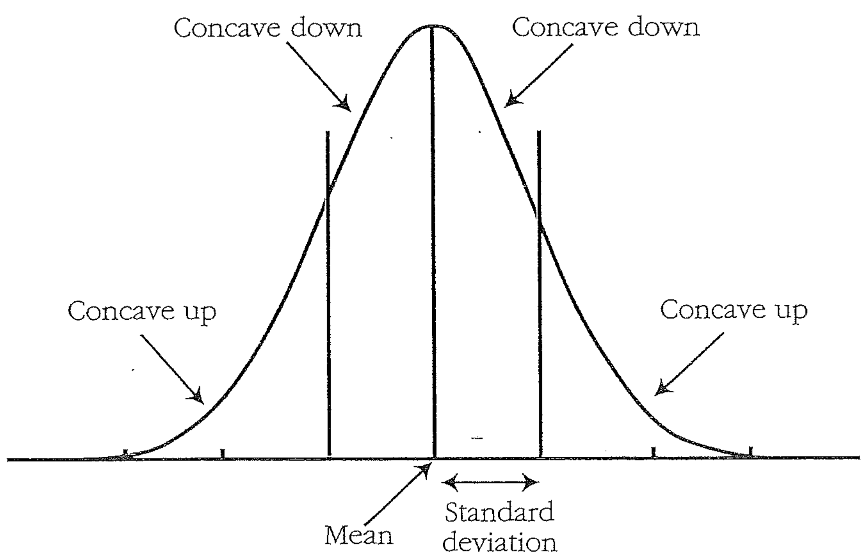
Note: In order to understand exactly where the specific numbers "68%" and "95%" come from, you would need to have a precise definition of *normal distribution,* a definition that is stated using concepts from calculus.

Geometric Interpretation of Standard Deviation

Geometrically, the standard deviation for a normal distribution turns out to be the horizontal distance from the mean to the place on the curve where the curve changes from being concave down to concave up.

In the diagram "Visualizing the Standard Deviation," the center section of the curve, near the mean, is concave down, and the two "tails" (that is, the portions farther from the mean) are concave up.

The two places where the curve changes its concavity, marked by the vertical lines, are exactly one standard deviation from the mean, measured horizontally.

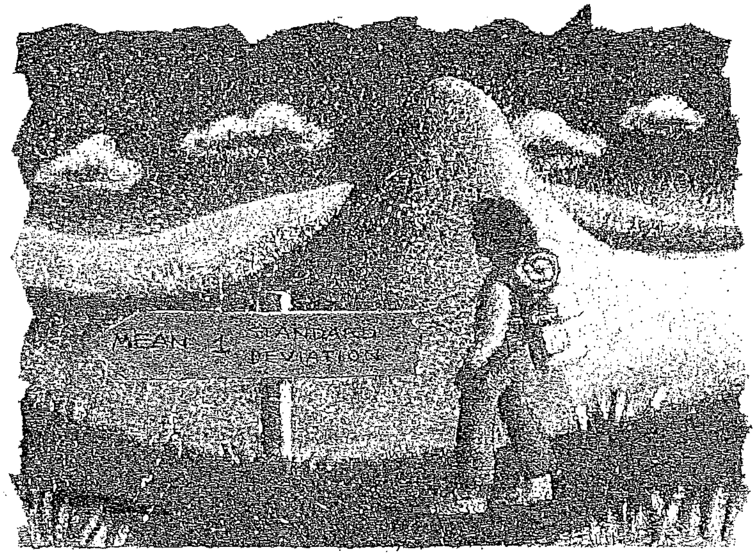


Visualizing the Standard Deviation

Homework 12 The Best Spread

Here are the four sets of data from *Data Spread.*

Set A	19, 19, 20, 20, 21, 21
Set B	10, 10, 20, 20, 30, 30
Set C	12, 13, 13, 27, 27, 28
Set D	9, 20, 20, 20, 20, 31



1. Write down the way you arranged the four sets of data in that assignment, from the set that is *least spread out from the mean* to the set that is *most spread out from the mean.*

2. a. Calculate the standard deviation of each set of data.

   b. Use your answers from Question 2a to arrange the four sets of data from the set with the smallest standard deviation to the set with the largest standard deviation.

3. a. Are the two arrangements, from Questions 1 and 2b, the same?

   b. If the arrangements are not the same, explain the reasoning you used in your arrangement from Question 1.

4. Recall that Rinky thought that *range* was a good method for measuring data spread. Make up two new sets of data, set X and set Y, in which set X has a larger standard deviation than set Y but set Y has a larger range than set X.

Making Friends with Standard Deviation

You will be working with the concept of standard deviation in connection with the unit problem throughout the rest of the unit. Before you begin that work, it will be helpful for you to gain some more familiarity with the concept.

1. First, explore what happens to the mean and the standard deviation of a set of data when you add the same number to each member in the set.

   a. As a group, make up a set of five numbers that are all different. Find the mean and the standard deviation of your set.

   b. Now choose a nonzero number and add it to each member of your set. Find the mean and the standard deviation of your new set.

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c. Repeat Question 1b, using a different nonzero number. Add this number to each member of your original set of data, and find the mean and standard deviation of the new set. Keep repeating this process until you see a pattern, and then describe that pattern.

d. Explain why your pattern should occur.

- Explain why the mean changes as it does when you add the same thing to each member of the set.
- Explain why the standard deviation changes as it does when you add the same thing to each member of the set.

2. Now explore what happens to the mean and the standard deviation of a set of data when you multiply each member in the set by the same number.

a. Begin with the same set of data as in Question 1a. Then choose a nonzero number other than 1. Multiply each member of your set by that number and find the mean and the standard deviation of the new set.

b. Choose another nonzero number other than 1, and repeat what you did in part a.

c. Keep choosing new nonzero numbers to use as multipliers for each member in your set. Find the mean and the standard deviation of each new set, until you see a pattern. Describe that pattern.

d. Explain why your pattern occurs.

3. Make up a set of data for each of these pairs of conditions.

a. Mean, 6; standard deviation, 1

b. Mean, 10; standard deviation, 1

c. Mean, 7; standard deviation, 2

Homework 13 Deviations

1. Find the mean and the standard deviation of this set of data.

$$24, 25, 15, 19, 17$$

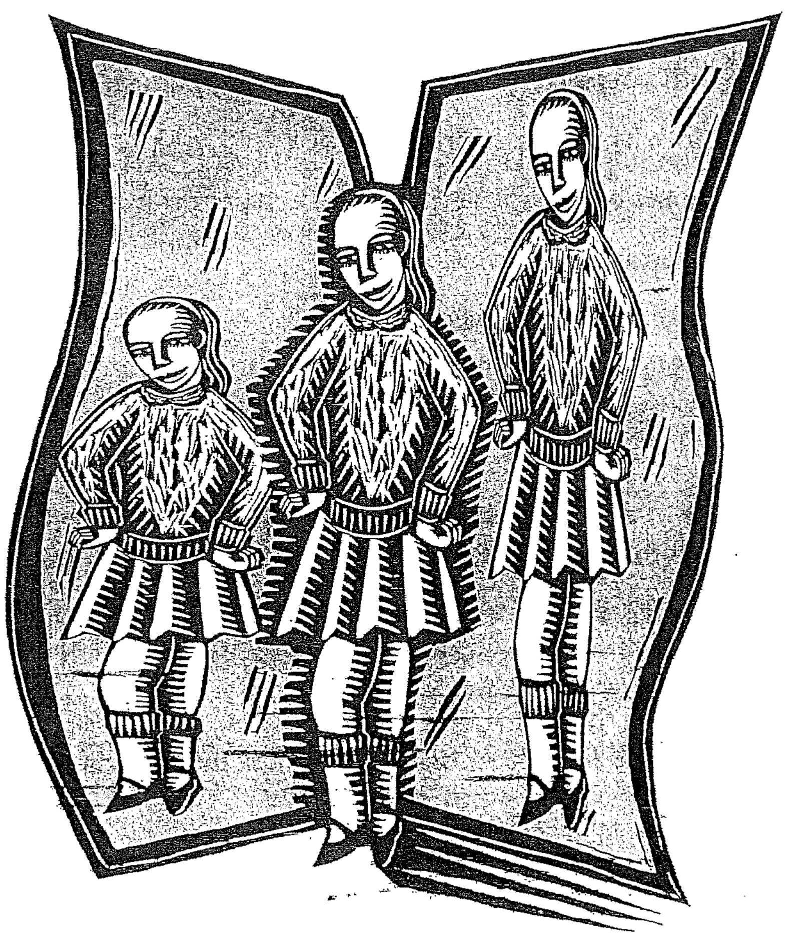
Your task in the rest of this assignment is to make up new sets of data items, each having either the same mean or the same standard deviation as the data set in Question 1.

If you can, do these problems without actually calculating the mean or the standard deviation of each new set of data, and explain how you know without calculating that the data set fits the conditions.

2. Make up a set of five data items that has the *same mean* as the data set in Question 1 but has a *smaller standard deviation*.

3. Make up a set of five data items that has the *same mean* as the data set in Question 1 but has a *larger standard deviation*.

4. Make up a set of five data items that has the *same standard deviation* as the data set in Question 1 but has a *different mean*.



Homework 14 Penny Weight Revisited

In *Homework 9: Penny Weight,* you saw that Sarah and Tom had been weighing a bunch of pennies on a sensitive scale. For your convenience, here again are the results that they got (in milligrams).

2600	2604	2607	2610	2612	2615	2616
2617	2618	2619	2623	2623	2624	2625
2626	2627	2630	2631	2636	2637	

1. Compute the mean and standard deviation of these weights. Record all your computations clearly so you can compare results with others in your group.

   Remember the steps in finding the standard deviation.

   a. Find the mean.

   b. Find the difference between each data item and the mean.

   c. Square each of the differences.

   d. Find the average (mean) of these squared differences.

   e. Take the square root of this average.

2. Now reconsider the problem of the penny that Sarah's and Tom's Uncle Jack claimed was counterfeit. When Sarah and Tom weighed that penny, they got a weight of 2641 milligrams.

   a. Based on your results in Question 1, what can you say about the probability that a real penny would have a weight so far from the mean?

   b. Do you think that Uncle Jack's penny is real or counterfeit?

Homework 15 Can Your Calculator Pass This Soft Drink Test?

1. A soft drink company sells its beverage in one-liter bottles. (*Reminder:* A liter is equal to 1000 milliliters. The abbreviation for millileter is mL.)

   The machine that fills the bottles is not perfect. The amount of soft drink it puts into the bottles fits a normal distribution, with a mean of 1000 mL (fortunately) and a standard deviation of 5 mL.

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If the filling machine puts more than 1005 mL into a bottle, the bottle will very likely spill when opened, causing customers to complain. If the machine puts less than 995 mL into a bottle, the amount in the bottle will be visibly less than it should be, causing customers to feel cheated.

A quality-control worker checks the bottles after they are filled and before they are sealed to see if they fit within the bounds of these conditions. If a bottle is either too full or not full enough, the worker removes the bottle from the assembly line to be corrected.

Approximately what percentage of bottles get removed from the assembly line?

2. A manufacturer of graphing calculators keeps track of the length of time it takes before the product is returned for repair. She finds that the mean is 985 days and the standard deviation is 83 days.

   She wants to set a time period during which her company will warranty the calculators—that is, a period in which they will replace them at no cost to the customer if the products do not function properly. She does not want to have to replace more than 2.5% of those sold.

   Assume that the number of days before calculators need repair is normally distributed. How many days' warranty would you advise her to give her customers? Explain your reasoning.

3. Students' scores on a certain college entrance examination follow a normal distribution, with a mean of 490 and a standard deviation of 120. The college of your choice, Big State University, accepts only students whose scores on this test are 600 or higher.

   Estimate the percentage of students who are eligible for admission to Big State on the basis of their test scores, and explain your reasoning.